PBK-0030-491002 Seat No. ________

B. Sc. / M. Sc. (Applied Physics) (Sem. I) (CBCS) Examination

November / December - 2018

Paper - II : Fundamentals of Mathematics

(New Course)

Faculty Code : 0030
Subject Code : 491002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions: (1) All questions are compulsory

(2) Numbers in the right margin indicate marks

1 Attempt any SEVEN short questions (Two marks each) **14**

1. If $A=\begin{bmatrix}1 & 2\\ 3 & -1\end{bmatrix}$ and $B=\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}$ then find AB.
2. Expand: $\begin{vmatrix}1 & 2 & -3\\ -2 & 0 & 4\\ -5 & 3 & 6\end{vmatrix}$
3. Define: Transpose of a matrix.
4. Find the polar form of $1-i$.
5. Divide $1+i$ by $1+2i$.
6. Evaluate: $\int x\log x dx$.
7. Evaluate: $\lim\limits_{x\to o}\dfrac{3^x+\sin x-1}{x}$
8. Find $\dfrac{dy}{dx}$, if $y=\dfrac{\tan x}{x}$.

9. If $A = J + K - I$ and $B = 2I + J - 3K$ then find $|A \times B|$.

10. If $A = (2, -3, 5)$ and $B = (11, -6, -8)$ then prove that A is perpendicular to B.

2 (A) Write answers of any TWO (Five marks each) **10**

1. If $A = \begin{bmatrix} 1 & 3 & 0 \\ -1 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 3 \\ -1 & 1 & 2 \end{bmatrix}$ then verify that $(AB)^T = B^T \cdot A^T$

2. Find the inverse of the matrix $\begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$ by row operation.

3. Solve $x + y + 2z = 9;\ 4x - 2y + 6z = 8;\ 3x + y - 3z = 13$ by matrix method.

4. Reduce matrix $A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ into its normal form and hence find its rank.

(B) Write answers of any TWO (Two marks each) **04**

1. Define: Singular matrix.
2. Define: Orthogonal matrix.
3. If $A = \begin{bmatrix} 1 & -3 \\ 5 & 6 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 \\ 0 & -2 \end{bmatrix}$ then find $(A + B)^T$.
4. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ then find $A^2 + 3I$.

3 (A) Write answers of any TWO (Five marks each) **10**

1. Express the following complex numbers in polar form. Also find modulus and principal argument

   $a)\, 1+i$ $\qquad$ $b)\, \sqrt{3}+i$.

2. If $2\cos\theta = x+\frac{1}{x}$, prove that $2\cos r\theta = x^r + \frac{1}{x^r}$.

3. Find the complex number z if $\arg(z+1)=\frac{\pi}{6}$ and $\arg(z-1)=\frac{2\pi}{3}$.

4. State and prove De Moivre's theorem.

(B) Write answers of any TWO (Two marks each) **04**

1. Find the square root of $z = 1+\sqrt{3i}$.

2. Express the $z = \frac{2-\sqrt{3i}}{1+i}$ in the form $x+iy$.

3. Simplify: $\frac{(\cos 3\theta + i\sin 3\theta)^4 (\cos 4\theta - i\sin 4\theta)^5}{(\cos 4\theta + i\sin 4\theta)^3 (\cos 5\theta + i\sin 5\theta)^{-4}}$.

4. Find the real values of x, y from the $-3+ix^2y = x^2+y-4i$.

4 (A) Write answers of any TWO (Five marks each) **10**

1. Evaluate:

   $a)\, \lim_{x\to 2} \frac{x^2+x-6}{x^2-4}$ $\qquad$ $b)\, \lim_{x\to 0} \frac{2x+3\sin x}{\tan x + 4x}$.

2. Find $\frac{dy}{dx}$, if $y = (\cos x)^x$.

3. Evaluate:

   $a)\, \int \frac{e^x (x+1)}{\cos^2 (xe^x)} dx$ $\qquad$ $b)\, \int \frac{x^4+x^2+1}{x^2+1} dx$.

4. Evaluate: $\int_0^{\frac{\pi}{2}} x\sqrt{2x-x^2}\, dx$ using reduction formula

(B) Write answers of any TWO (Two marks each) **04**

1. Evaluate: $\lim_{x \to \infty} \frac{x^3 + 2x^2 + 1}{x(x^2 + 1)}$.

2. Find $\frac{dy}{dx}$, if $y = x^a \cdot a^x$.

3. Evaluate: $\int e^x \cdot \sin e^x dx$

4. Define: Odd and Even function

5 (A) Write answers of any TWO (Five marks each) **10**

1. Find the angle between the vectors $A = i + 2j + 3k$ and $B = -2i + 3j + k$.

2. If $p = 9j - 4i + 6k$, $q = 7j + 10k$ and $r = 6j + 6k - i$ then prove that $(p - r) \cdot (q - r) = 0$.

3. If $a = 2i - 3j + 4k$ and $b = i - j + k$ then find perpendicular unit vector to both $a + b$ and $a - b$.

4. Find $curlF$ and $divF$, where $F = grad(x^3 + y^3 + z^3 - 3xyz)$.

(B) Write answers of any TWO (Two marks each) **04**

1. If $a = 3i - j - 4k$, $b = -2i + 4j - 3k$ and $c = i - 2j - k$ then find $a \cdot (b + c)$.

2. If $a = i - 2j$ and $b = -i + j + 3k$ then find direction cosines of $3a + 2b$.

3. Define: Vector.

4. Define: Unit vector and Modulus of vector.
